

Effective Strategies for Students with Learning Disabilities in Algebra

LeeAnn Calvert

Lynchburg College

Learning disability as defined by Pullen, Lane, Ashworth, and Lovelace (2011) is a term used to explain the struggle that a student of average intelligence faces with basic academic or functional skills. “Learning disabilities can affect a person’s ability to understand or use spoken or written language, do mathematical calculations, coordinate movements, or direct attention” (Pullen, et al., 2011, p. 187). Learning disabilities are the predominant disability in special education in the United States with half of the special education population receiving services with a learning disability label (Denton, Vaughn, & Fletcher, 2003).

The reauthorization of the Individuals with Disabilities Education Improvement Act (IDEA) (2004) has required that students with disabilities meet the same math standard by grade level as their peers (Scheuermann, Deshler, & Schumaker, 2009). Additionally, Virginia SOL and common core standards require that all students successfully complete course requirements and end of course assessment, and algebra is now the lowest class in the high school course sequence (VDOE, 2012). Meeting these mathematical standards presents a challenge for any student who struggles in mathematics; however, it may be an overwhelming task for a student with a disability (Scheuermann et. al., 2009).

Due to these new requirements educators are seeking methods to effectively instruct students with learning disabilities in mathematics. Researchers have found that the current trend of educating students with a learning disability in the general education classroom using traditional methods is simply not working (Scheuermann et al., 2009). Deshler, Lenz, Bulge, Schumacher, Davis, and Grossen (2004) reported that students with learning disabilities plateau in mathematics achievement as they reach upper middle and high school grades, seven through twelve, when these traditional methods are used.

It has now become crucial to develop strategies and methods to effectively educate students with learning disabilities in mathematics. Strategies that target and improve student's problem solving ability should positively impact their overall math performance, grades, graduation rates, and postsecondary success (Montague, Enders, & Deitz, 2011). This paper will review research-based practices in algebra employed to successfully educate upper middle and high school students with learning disabilities and provide a summary of those practices which are effective and useful.

**Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice, 18*(2), 121-131.**

In the introduction of the first article reviewed, Witzel, Mercer, and Miller (2003) explained that students struggle with algebra predominantly because it is an abstract concept. In order to successfully instruct students with learning disabilities in algebra, abstract mathematical concepts must be broken down into concrete steps. A possible solution to this problem is the concrete – to representational – to abstract (CRA) sequence of instruction. The process starts with the use of manipulatives to solve mathematical problems and moves to pictorial representations once a student is successful with the concrete manipulatives. Witzel et al. (2003) stated that the difficulty lies in building a concrete representation of an algebraic concept that keeps the abstract description of the concept. In other words, if a model is too simple and contains broad generalizations it will only work for simple problems. As problems become more complex the model will fail to work which will actually hinder the learning process. The purpose of this study was to examine a revised and explicit CRA model designed for complex algebra equations to see if this is an effective model for students with learning disabilities.

Witzel et al. (2003) used a pre-post follow up design with random assignments of clusters. Students were divided into a comparison and treatment group and clustered by classroom. This study used 12 teachers, 10 classrooms, and 358 sixth and seventh grade students. There were 34 students in the treatment group that were one to one matched with 34 students in the comparison group. The students in the treatment group received CRA instruction, and the students in the comparison group received abstract only, traditional methods. The students in both the treatment and comparison group had a documented disability in mathematics with math goals listed in their IEP or qualified for at risk for math difficulties by meeting set criteria that included achievement testing scores. These 34 pairs of students were matched according to state math achievement testing, teacher, and previous math course. To account for the many differences in teachers, the 34 student pairs were matched across teacher and type of instruction (Witzel et al., 2003).

This research was a strong pretest- posttest control group design in which students were randomly assigned to the control and experimental groups. One-to-one matching of the research participants added additional strength and validity to the research design. Thorough and detailed instruction was given to teachers on how to implement the CRA model. Witzel et al. (2003) also used a fidelity check to safeguard that the sequence and structure of instructional elements was implemented correctly. Data analysis consisted of performing repeated measures analysis of variance on CRA versus abstract and pretest, posttest, and follow up. Selection of participants, pre-post follow up testing, and data analysis was thoroughly described by researchers. The research methods used were strong and results could be determined as valid practice to implement for students with learning disabilities.

Witzel et al. (2003) results indicated that although both the treatment and comparison group improved significantly from pretest to posttest, the treatment group that received the CRA instruction out performed students in the comparison group that only received traditional instruction. The students in the treatment group also made fewer mistakes using negative numbers and with transforming equations before solving.

One implication of this study for students with learning disabilities is that approaches such as CRA that contain concrete and pictorial representations of abstract concepts will promote understanding and facilitate student success. Although manipulatives have been predominantly used in the elementary setting, this study finds that they are a valuable tool for instruction in the secondary setting as well (Witzel, et al., 2003). The CRA algebra model demonstrated by Witzel et al. (2003) generalizes abstract concepts in a manner that facilitates students' active engagement as it adapts to each student's learning style. This research provides a worthy contribution to the understanding of how to best educate algebra students with learning disabilities.

**Scheuermann, A. M., Deshler, D. D., & Schumaker, J. B. (2009). The effects of the explicit inquiry routine on the performance of students with learning disabilities on one-variable equations. *Learning Disability Quarterly*, 32(2), 103-120.**

Scheuermann et al. (2009) also used the CRA instructional sequence; their literature review found that when students concretely experience a mathematical concept, they are able to truly understand the concept being introduced. Past research also revealed that students with learning disabilities improve performance when explicit and sequential instruction is used. Additionally, previous research led Scheuermann et al. (2009) to the conclusion that sequential mathematical scaffolding was also an essential component of the development of mathematical

proficiency and understanding. Although each practice was evidence-based, this combination of educational practices from general education and special education had not been created (Scheuermann et al., 2009).

The purpose of the research conducted by Scheuermann et al. (2009) was to investigate and test an instructional method that incorporated critical elements of both general education and special education into one mode of instruction. These critical elements included: inquiry, dialog, manipulative, CRA and explicit instruction. Scheuermann et al. termed this combined approach Explicit Inquiry Routine (EIR). EIR contained three main parts for implementation: content that was presented sequentially and explicitly, learning that was scaffolded building on previous knowledge base, and a systematic approach to using various modes of illustration. CRA sequence of instruction was used to accomplish the various modes of illustration.

There were 14 participants in this study in grades sixth through eighth. Each of the participants was identified with the following criteria: diagnosed with a learning disability, IQ standard score above 85, math achievement test score below 25 percentile, and pre-test measure of one variable equation score less than 50% accuracy (Scheuermann et al., 2009). There were four measurement instruments used, which included a word problem test, a concrete manipulation test, a Far-Generalization Test, and the Key Math-R. Each test had a specific purpose and was implemented to address specific questions. For example, the purpose of the word problem test might be to interpret if students had learned to solve simple one-variable equations, and the question under investigation might be; does this improve their ability to solve more complex (untaught) one-variable equations? Explanation of data analysis and data collection was detailed and comprehensive. Each assessment tied directly to a component of the EIR implementation.

Scheuermann et al. (2009) indicated that by using the EIR approach students who have learning disabilities can not only solve one-variable equations but can transfer this learning into a variety of situations successfully and retain the learning up to 11 weeks. These findings support the use of clear, detailed, and sequential instruction termed explicit instruction to improve mathematical performance. This research expands the existing research by showing that inquiry and dialog combined with explicit instruction can enhance learning for students with learning disabilities (Scheuermann et al., 2009). It also provides additional support for the fact that the CRA instructional sequence successfully engages students in concrete and representation activities that form a strong and necessary bridge to abstract concepts.

The findings of the study conducted by Scheuermann et al. (2009) are limited however because of weak research design. The study did not use random assignments of groups or a control group, and failed to compare EIR with any other instructional method. Since the participants were selected from a private school for students with learning disabilities, this population may not accurately represent a public school setting of heterogeneously groups students.

**Ives, B. (2007). Graphic organizers applied to secondary algebra instruction for students with learning disorders. *Learning Disabilities Research & Practice*, 22(2), 110-118.**

Another form of instruction applied to students with learning disabilities to improve the understanding of algebra is the use of graphic organizers. Ives (2007) explained that research indicates a clear link between some language disabilities and mathematics disabilities. Such a link suggests that students would benefit from instruction in mathematics that is not dependent on a student's language skills. An example reviewed by Ives (2007) of an instructional method

that is less dependent on reading and comprehension is Concrete-Semi-concrete-Abstract (CSA) teaching sequence. While research finds this technique to be very useful, the technique lacks the ability to be applied to higher level mathematical concepts which do not easily fit into a concrete model. Ives (2007) presents the concept of graphic organizers as a solution to assist struggling students understand concepts and relationships presented in mathematics. Graphic organizers were previously used to aide students in reading comprehension, but they can be applied to mathematical symbols, expressions, and equations at any level of complexity.

Ives (2007) compared two groups using experimental design with random assignment of participants. The control group received instruction on solving linear equations by implementing direct and strategic instruction. The treatment group also received explanation on how to solve linear equations using direct and strategic instruction with the addition of graphic organizers. Participants were chosen from a private school in Georgia for sixth through twelve grade students with learning disabilities or attention disorders. After the initial investigation, Ives (2007) also conducted a systematic replication of the study using graphic organizers with a different set of participants and a different mathematical skill.

This study used a strong experimental research design that included random assignment to groups, a control group, and follow up study for systematic replication. The author explained the research question and progression of the investigation in a clear and logical manner. Therefore, this study has a strong design and structure making its implications relevant to the instruction of students with learning disabilities.

Research findings indicated that students who used graphic organizers developed a stronger understanding of the fundamental concepts of solving linear equations than students



who were not assigned to use graphic organizers (Ives, 2007). These results also support the idea that students with language disabilities are better served by non-verbal associations. The author, however, recognized in the limitations section that the effects of the intervention would decrease if implemented by classroom teachers rather than researchers. Ives (2007) concluded that a greater effect is shown if research studies implemented on students with learning disabilities have the treatment implemented by the researcher rather than the classroom teacher.

Additionally, Ives (2007) reported that more research is needed to further confirm the relationship between reading disabilities and calculation disabilities. This evidence would provide a crucial piece to drive mathematical instruction for students with learning disabilities.

**Montague, M., Enders, C., & Dietz, S. (2011). Effects of cognitive strategy instruction on math problem solving of middle school students with learning disabilities. *Learning Disability Quarterly*, 34(4), 262-272. doi: 10.1177/0731948711421762**

Montague et al. (2011) demonstrated that intervention effects change when they are brought into the classroom setting and implemented by teachers rather than researchers. The purpose of the research was to extend previous research on the *Solve It!* method by testing its effectiveness on students with learning disabilities in the regular classroom setting implemented by classroom teachers. Previous research intervention studies using the *Solve It!* method were conducted with students individually or in small groups by researchers or graduate assistants rather than in a classroom setting (Montague et al., 2011).

Montague et al. (2011) found in their literature review that cognitive strategy instruction works effectively and is beneficial to students with learning disabilities. Students with learning disabilities have not acquired the skills to allow them to effectively problem solve; they have

difficulty selecting the correct solution path and following through in the steps of execution. They also found that students with learning disabilities must be taught the process for problem solving explicitly; they must also be shown how to apply this process to word problems. Montague et al. (2011) explained that the explicit instruction model incorporates proven teaching strategies such as cueing, modeling, rehearsal, feedback, positive reinforcement, and overlearning. Explicit instruction, because of its interactive approach, allows teachers to adapt instruction to accommodate the needs of their students more effectively than direct instruction (Montague et al., 2011). *Solve It!* incorporates cognitive processes such as paraphrasing, visualizing, hypothesizing, estimation, and computation with self-regulation strategies such as self-questioning and self-monitoring to explicitly instruct students in problem solving.

Participants were chosen by matching 20 pairs of middle schools (Montague et al., 2011). Middle schools were matched on state test (FCAT) performance as well as socioeconomic status with one school from each matched pair being randomly assigned to the intervention condition. Unfortunately, because of attrition and training requirement prior to implementation, participant schools were reduced to eight intervention schools and sixteen comparison schools. The fact that randomization was affected by the attrition caused concern of internal validity. Math test items were selected from the *Solve It!* manual as the instrument of measurement. These problems were calibrated using item response theory methods to assign difficulty level. In addition one problem from the FCAT state assessment was included on each testing administration. Testing was implemented six times, the first of which was prior to the instruction to create baseline data and then testing was done monthly to assess progress.

Research results from Montague et al. (2011) indicated that eighth grade students across ability groups, including students with learning disabilities, showed a significant increase in

performance using *Solve It!* model of instruction than the comparison group who used traditional instruction. Additionally, students with learning disabilities out performed all ability groups' students who were in the comparison group by the end of the school year (Montague et al., 2011).

This research study was a strong experimental design including random assignments of groups. Ability matching that was conducted across schools and divisions provided strength and credibility to the research. The baseline data given on students prior to intervention implementation as well the use of a control group made discussion of the results clear and significant. This article provides strong evidence once again for the use of explicit instruction for students with learning disabilities.

**Maccini, P. (1998). *Effects of an instructional strategy incorporating concrete problem representation on the introductory algebra performance of secondary students with learning disabilities* (Doctoral dissertation). Retrieved from <http://proquest.umi.com/arcane/lynchburg.edu:2048/pqdweb?SQ=Effects+of+instructional...>**

In her dissertation thesis, Maccini (1998) once again showed that students with learning disabilities struggle with higher order reasoning and problem solving skills, which are the main component of algebra instruction. Maccini (1998) believes that in order to understand how to successfully assist students with learning disabilities one must first understand the basis for how students achieve in algebra. In order to perform successfully in algebra, a student must understand “the following components: basic skills and terminology, problem representation, problem solution, and self- monitoring strategies” (Maccini, 1998, p.2). Her literature review

found that effective problem solving includes cognitive (procedural) steps and metacognitive (self-monitoring) steps. Maccini (1998) found that students with learning disabilities are vastly different from proficient problem solvers. Students with learning disabilities have difficulty selecting correct operations, visualizing problem situations, using appropriate strategies, and monitoring themselves.

After the examination of results from previous studies, Maccini (1998) found the following instructional strategies to be valuable for students with learning disabilities: concrete, semi-concrete, and abstract instructional sequence (CSA), self-monitoring strategies, common problem solving strategies, and a mnemonic device to assist students with strategy steps; all of these were implemented into the research investigation. Outcomes of the research investigation were produced by a single subject design along with qualitative data. The purpose of the research was to address each of the following research questions: will student's ability to symbolize and solve word problems using integers significantly improve when provided strategic instruction at the CSA levels; will students be able to generalize these treatment effects into new and more complex problems; will students retain the treatment effects over time.

There were six participants selected who met the following conditions: diagnosed with a learning disability, currently enrolled in algebra, received assistance in the resource room for algebra, mainstreamed in general education classroom, and consistently scored below 80% on baseline test on problem solving with integers. Maccini (1998) was very thorough in the description of each participant including grade, age, socio-economic status, ethnicity, and scores on standardized test such as the Woodcock Johnson applied math problems and calculations.

A detailed explanation was given by Maccini (1998) of data collection and scoring guidelines for data analysis. Measurement included percent correct on problem representation, percent correct on problem solution and answer, and percent of strategy used in process for each of the three phases' concrete, semi-concrete and abstract applications. Audio tapes were used to record verbalization data during problem solution and representation.

Results of the investigation indicated that secondary students with learning disabilities dramatically improved in their ability to correctly solve problems using integers. Additionally, their use of instructional strategies over time also increased, and they could generalize the treatment effect to more complex and new problems. They were also able to retain this treatment effect over time for the multiplication and division of integers (Maccini, 1998).

This dissertation conducted by Maccini (1998) was well-structured, easy to read and understand, and provided valuable information relevant to the success of students with learning disabilities in mathematics. The research question and progression of the investigation was explained in a clear and logical manner. The analysis was detailed, and the findings were directly related to the research questions. However, it is hard to generalize the effects of treatment to the secondary setting because of the limited sample size. Additionally it was hard to distinguish whether all the treatments applied in this multi treatment approach were effective or necessary.

### **Discussion**

Several themes of effective practice to educate students with learning disabilities in algebra emerged in through this investigation. The first was the consistent use of models/techniques that give abstract concepts a concrete representation. Witzel et al. (2003),

Scheuermann et al. (2009), and Ives (2007) reported that when student concretely experience an abstract concept they have a better understanding of the process being taught. These interactive systematic procedures such as CRA facilitate student understanding and success. Students who were taught using CRA methods not only had improved performance but were able to transfer the learning to other equation types and retain the information over time. Models that work in elementary schools often however do not work as mathematics becomes more complex in high school courses. Caution must be taken to make sure that models used are not overly simplistic and apply to only a few problems. If the model fails when problems become more difficult it will actually hinder the learning process (Witzel et al. 2003; Ives, 2007). To accommodate complex mathematics, Ives (2007) suggested that graphic organizers be used to bridge this critical link from abstract concepts to concrete representation. Graphic organizers also increase student performance and provided information retention and transference. Despite the debate over which method is best practice, it is obvious that students with disabilities are better served by using tools such as CRA and graphic organizers that generalize abstract concepts into concrete representations using an interactive approach to instruction.

Another interactive approach to mathematics that has proven success in students with learning disabilities is explicit instruction that contains an element of self-monitoring and regulation. Students with learning disabilities do not have the ability to effectively problem solve they must be shown explicitly the step by step process to obtain a solution. The modeling, cueing, rehearsing, reinforcement and overlearning found in explicit instruction enables students to be successful (Montague et al., 2011). Students with learning disabilities have problems selecting the next step, visualizing the problem, using appropriate strategies and monitoring their progress. Therefore, it is crucial that they be given procedural steps as well as self-monitoring

steps to facilitate success (Maccini, 1998). Treatments that use explicit instruction combined with self –monitoring will assist students with learning disabilities in comprehension as well as retention of information over time.

Additionally, through this review of effective mathematics instruction for students with learning disabilities, it was suggested that there is a link between reading disabilities and mathematic disabilities. This is why non- verbal associations such as CRA and graphic organizers, which were originally used to aide in reading comprehension, are so successful for students who have learning disabilities in mathematics (Ives, 2007). Ives (2007) stated that effective mathematics instruction for students with learning disabilities would consist of instruction that is not dependent on language skills. Unfortunately, he stated that there is insufficient evidence in research to confirm this link between reading disabilities and mathematics disabilities. Continued research is needed to clearly outline the link between reading disabilities and mathematics disabilities. This critical piece would allow us to further assist students with learning disabilities in mathematics.

Teachers and administrators must realize that students with learning disabilities struggle with problem solving and higher order reasoning, which is a main component of Algebra (Maccini, 1998). A combination of special education techniques such as CRA, self-monitoring, and explicit instruction must be combined with general education techniques of inquiry, dialog, and the use of manipulatives in order to produce effective instruction for students with learning disabilities (Scheuermann et al., 2009). It has therefore become crucial for special educators and general educators to communicate, collaborate, and collectively plan for the instruction of students with learning disabilities that are being services in the general education setting. By using collaboration, the general educator can provide expertise into content and structure of the

course and the special educator can provides expertise in how the curriculum can be adapted to meet each student's unique learning needs (Cook, McDuffie- Landrum, Oshita, & Cook, 2011). Expertise of both specialized teachers is needed to enable students with learning disabilities in mathematics to be successful. As administrators, we must make sure that this collaboration is taking place. We must schedule time into the master schedule to allow time for special educators to collaborate or co-teach with general educators.

It is also important as leaders to realize that results change when strategies are implemented by teachers in a classroom setting rather than researchers (Montague et al., 2011). Results of the studies reviewed in this investigation may not have the same impact when implemented. Proper training, monitoring and follow through are required to successfully implement a new tool or teaching strategy. Treatment fidelity as well as the use of effective procedures are both key components to achieve results from implementation. As leaders it is our job to insure that all of those components are present, monitored, and evaluated for effectiveness.



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